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USE OF COVARIATES TO EXPLAIN (OR PREDICT) LIFE LENGTH.(U)
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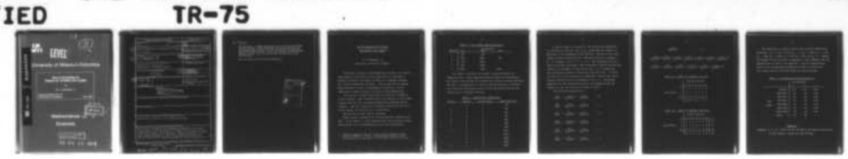

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Use of Covariates to Explain (or Predict) Life Length

by

W. A. Thompson, Jr.

Technical Report No. 75
Department of Statistics

July 1978

Mathematical Sciences

78 09 13 024

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 78-1	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER 9
4. TITLE (and Subtitle) 6 Use of Covariates to Explain (or Predict) Life Length.		5. TYPE OF REPORT & PERIOD COVERED Interim Technical Report,
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) 10 W. A. Thompson, Jr		8. CONTRACT OR GRANT NUMBER(s) 15 N00014-75-C-0443 (NR042-282
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Statistics University of Missouri-Columbia		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS 12 8p.		12. REPORT DATE 11 1 July 1978
		13. NUMBER OF PAGES 5
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) 14 TR-75, 78-1		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

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17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Reliability, turbines, life length, covariates

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In medical, biological and engineering research, test results often appear in the form of a life table and it is desired to explain or predict life length in terms of, or perhaps allowing for, certain explanatory variables. Thus we may wish to explain human longevity in terms of geographical location or time of remission of Leukemia patients in terms of drug admin-

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S/N 0102-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

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How can data of this kind be analyzed?

USE OF COVARIATES TO EXPLAIN (OR PREDICT) LIFE LENGTH [1]

W. A. Thompson, Jr.

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Table 1 gives a minature set of fictitious but representative data. In the table, a + sign indicates that the particular turbine installation was still operating when the data was analysed.

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Table 1 - Life length (operating hours)

Turbine	installation 1st	2nd	3rd
1	643	970+	
2	860	1800	49+
3	792	1200+	
4	600	1430+	
5	1004	880	1500+

This report illustrates an analysis using the methods of Thompson (1977). Since improved design features are incorporated, the various installations of a turbine are treated as being unrelated. The data is reorganized as in Table 2 and the time axis is divided into intervals sufficiently short so that individual failures and losses occur in separate intervals.

Table 2 - Ordered life length data

failure	Turbine	Installation	Operating hours
	2	3	49+
a	4	1	600
b	1	1	643
c	3	1	792
d	2	1	860
e	5	2	880
	1	2	970+
f	5	1	1004
	3	2	1200+
	4	2	1430+
	5	3	1500+
g	2	2	1800

A logistic model is assumed for the conditional probability of surviving an interval $[t_j, t_{j+1})$ given that the turbine is operating at the beginning of the interval. Specificly the conditional survival probability for a $k^{\underline{th}}$ installation is assumed to be $[1 + \exp(\beta_k + \eta_j)]^{-1}$; $k = 1, 2, 3$. The parameters are estimated by maximum likelihood. Only differences between installation effects are estimable; we arbitrarily take $\beta_1 = 0$, measuring the installation effects as deviations from the first. It turns out that for a time interval not containing a failure the corresponding interval effect is estimated as $\hat{\eta}_j = -\infty$. Likelihood equations for the remaining parameters are prepared using the numbers at risk and surviving as given in Table 3.

The likelihood equations are:

$$\frac{5}{1+e^{\eta_a}} + \frac{5}{1+e^{\beta_2+\eta_a}} + \frac{1}{1+e^{\beta_3+\eta_a}} = 10$$

$$\frac{4}{1+e^{\eta_b}} + \frac{5}{1+e^{\beta_2+\eta_b}} + \frac{1}{1+e^{\beta_3\ \eta_b}} = 9$$

$$\frac{3}{1+e^{\eta_c}} + \frac{5}{1+e^{\beta_2+\eta_c}} + \frac{1}{1+e^{\beta_3+\eta_c}} = 8$$

$$\frac{2}{1+e^{\eta_d}} + \frac{5}{1+e^{\beta_2+\eta_d}} + \frac{1}{1+e^{\beta_3+\eta_d}} = 7$$

$$\frac{1}{1+e^{\eta_e}} + \frac{5}{1+e^{\beta_2+\eta_e}} + \frac{1}{1+e^{\beta_3+\eta_e}} = 6$$

$$\frac{1}{1+e^{\eta_f}} + \frac{1}{1+e^{\beta_2+\eta_f}} + \frac{1}{1+e^{\beta_3+\eta_f}} = 4$$

$$\frac{1}{1+e^{\beta_2+\eta_g}} = 0$$

$$\frac{5}{1+e^{\beta_2+\eta_a}} + \frac{5}{1+e^{\beta_2+\eta_b}} + \frac{5}{1+e^{\beta_2+\eta_c}} + \frac{5}{1+e^{\beta_2+\eta_d}} + \frac{5}{1+e^{\beta_2+\eta_e}} + \frac{3}{1+e^{\beta_2+\eta_f}} + \frac{1}{1+e^{\beta_2+\eta_g}} = 27$$

$$\frac{1}{1+e^{\beta_3+\eta_a}} + \frac{1}{1+e^{\beta_3+\eta_b}} + \frac{1}{1+e^{\beta_3+\eta_c}} + \frac{1}{1+e^{\beta_3+\eta_d}} + \frac{1}{1+e^{\beta_3+\eta_e}} + \frac{1}{1+e^{\beta_3+\eta_f}} = 6$$

Table 3 a - Number of turbine's at risk

		failure interval						
		a	b	c	d	e	f	g
	1	5	4	3	2	1	1	0
installation	2	5	5	5	5	5	3	1
	3	1	1	1	1	1	1	0
		11	10	9	8	7	5	1

Table 3 b - Number of turbines surviving

		failure interval							
		a	b	c	d	e	f	g	
	1	4	3	2	1	1	0	0	11
installation	2	5	5	5	5	4	3	0	27
	3	1	1	1	1	1	1	0	6
		11	9	8	7	6	4	0	44

The equations are solved theoretically and then numerically, obtaining: $\hat{\beta}_1 = 0$, $\hat{\beta}_2 = 3.156$, $\hat{\beta}_3 = -\infty$, $\hat{\eta}_a = -1.449$, $\hat{\eta}_b = -1.186$, $\hat{\eta}_c = -.833$, $\hat{\eta}_d = -.306$, $\hat{\eta}_e = .590$, $\hat{\eta}_f = .910$, and $\hat{\eta}_g = \infty$. Taking the lengths of all intervals to approach 0, the estimated survival probabilities of Table 4 are obtained. Survival probabilities here are the unconditional probabilities that a turbine will last longer than an indicated number of operating hours.

Table 4 - Estimated Survival Probability's

		installation		
		1	2	3
	[0,600)	1.00	1.00	1.00
	[600,643)	.81	.99	1.00
	[643,792)	.62	.98	1.00
time	[792,860)	.43	.96	1.00
range	[860,880)	.25	.93	1.00
	[880,1004)	.09	.86	1.00
	[1004,1800)	.03	.78	1.00
	[1800, -)	.00	.00	-

Reference

Thompson, W. A. Jr. (1977) "On the Treatment of Grouped Observations in Life Studies" Biometrics 33, 463-470.